Monday, March 6

Lab2 Solution Walkthrough

Lab2 Solution: Context Celebrity_c0

CONTEXT Celebrity_c0

CONSTANTS

k knows relation

c celebrity

P Set person

AXIOMS

axm1: $\boxed{P \subseteq \mathbb{N}}$

axm2: $c \in P$

axm3: $k \in (P \setminus \{c\}) \leftrightarrow P$

R3 axm4: $\boxed{\underline{k^{-1}[\{c\}]}} = P \setminus \{c\}$

axm5: $k \cap id = \varnothing$

END R1.

model persons via unique IDs.

inv.

the set of possible relations

the celebrity is known by everyone, except themselves

the set of persons by whom c is known

$(x, y) \in k$

↳ x knows y

C

$(y, x) \in k^{-1}$

↳ y is known by x

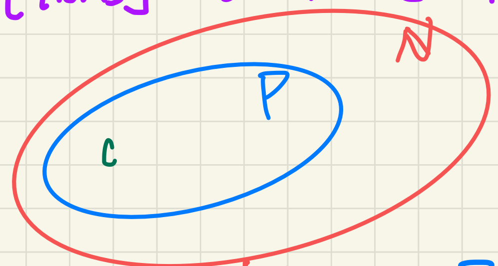
P = {Alan, Mark, Tom}

c = Tom

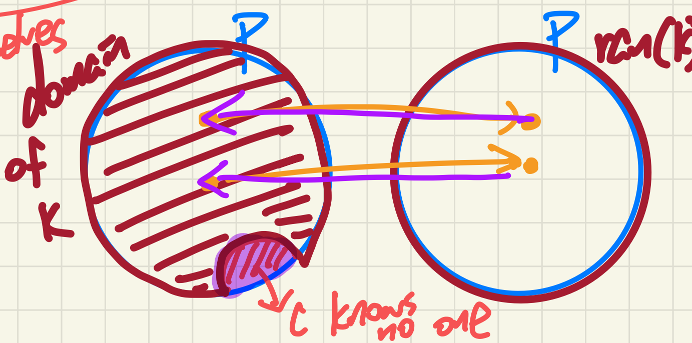
k = {(Alan, Mark), (Alan, Tom), (Mark, Tom)}

k^{-1} = {(Mark, Alan), (Tom, Alan), (Tom, Mark)}

k^{-1}[{Tom}] = {Alan, Mark} = P \ {Tom}



k^{-1}



Lab2 Solution: Machine Celebrity_1

May have to add extra constraints (which may be logically redundant) to guide the prover.

MACHINE Celebrity_1
SEES Celebrity_c0
VARIABLES
- r result of algorithm
- Q Set of potential Ce

INVARIANTS
- inv1: $r \in P$
  invariant from th
- inv2: $Q \subseteq P$
  new invariant: ev
- inv3: $c \in Q$
  new invariant: th

EVENTS
Initialisation
- **begin**
  - act1: $r :\in P$
  - act2: $Q := P$
- **end**

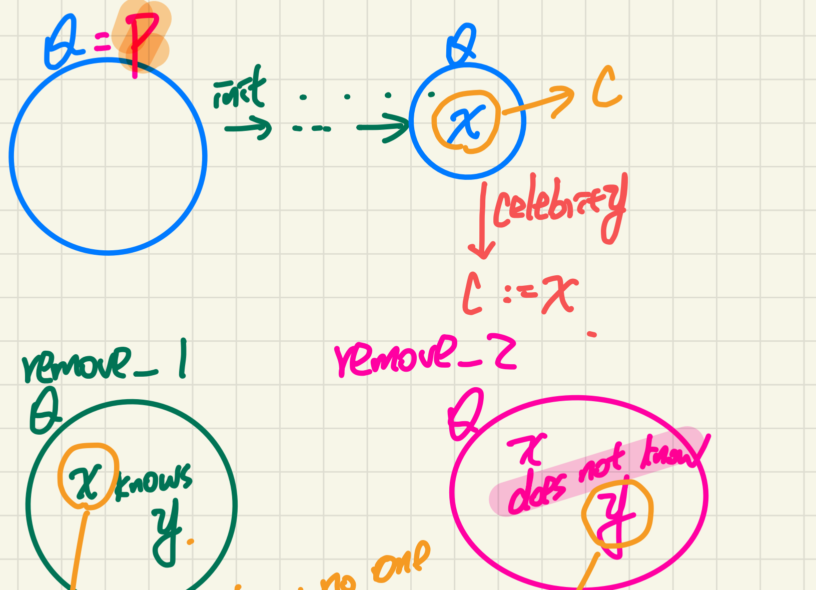
Event celebrity ⟨ordinary⟩ $\widehat{=}$
- **any**
  - x
- **where**
  - grd1: $x \in Q$
  - grd2: $Q = \{x\}$
- **then**
  - act1: $r := x$
- **end**

Event remove_1 ⟨ordinary⟩ $\widehat{=}$
- **any**
  - x
  - y
- **where**
  - grd1: $x \in Q$
  - grd2: $y \in Q$
  - grd3: $x \mapsto y \in k$
  - grd4: ⟨theorem⟩ $x \neq c$
    Without this guard,
    as a hypothesis in th
- **then**
  - act1: $Q := Q \setminus \{x\}$
- **end**

Event remove_2 ⟨ordinary⟩ $\widehat{=}$
- **any**
  - x
  - y
- **where**
  - grd1: $x \in Q$
  - grd2: $y \in Q$
  - grd3: $\boxed{x \mapsto y \notin k}$
  - grd4: $x \neq y$
  - grd5: ⟨theorem⟩ $y \neq c$
    Without this guard i
    as well as Q <: P as
- **then**
  - act1: $Q := Q \setminus \{y\}$
- **end**

Eg

P = {Alan, Mark, Tom}
c = Tom
k = {(Alan, Mark), (Alan, Tom), (Mark, Tom)}



x does not know no one ⇒ x cannot be the celebrity.

y not known by x ⇒ y can't be celeb.

MACHINE Celebrity_1
SEES Celebrity_c0
VARIABLES

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INVARIANTS

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EVENTS
Initialisation
begin
act1: $r :\in P$
act2: $Q := P$
end

Event $\boxed{\text{celebrity}}$ ⟨ordinary⟩ $\widehat{=}$
any
x
where
grd1: $x \in Q$
grd2: $Q = \{x\}$
then
act1: $r := x$ Tom
end

Event remove_1 ⟨ordinary⟩ $\widehat{=}$
any
x
y
where
grd1: $x \in Q$
grd2: $y \in Q$ t.
grd3: m. $x \mapsto y \in k$
grd4: ⟨theorem⟩ $x \neq c$
Without this guard,
as a hypothesis in th
then
act1: $Q := Q \setminus \{x\}$ m
end

Event remove_2 ⟨ordinary⟩ $\widehat{=}$
any
x
y
where
grd1: $x \in Q$
grd2: $y \in Q$
grd3: m a $x \mapsto y \notin k$
grd4: $x \neq y$
grd5: ⟨theorem⟩ $y \neq c$
Without this guard i
as well as Q <: P as
then
act1: $Q := Q \setminus \{y\}$
end

P = {Alan, Mark, Tom}
c = Tom
k = {(Alan, Mark), (Alan, Tom), (Mark, Tom)}

init
Q: Alan, Mark, Tom
remove_1
Q: Alan, Tom
remove_2 ∵ (Mak, Alan) ∈ k
∴ Alan can't be cel.
Q: Tom → the state where guards of remove_? are disabled.
celebrity
c := Tom

< init, remove_1, remove_2, celebrity >
one possible trace of algorithm

Another trace:
< init, remove_1, remove_2, celebrity >